## A REMARK ON NEUTRINO OSCILLATIONS OBSERVED IN Kamland Experiment

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## **Abstract**

It is demonstrated that the observation of neutrino oscillations in the atmospheric (K2K) neutrino experiments and unitarity of the mixing matrix implies that disappearance of the reactor  $\bar{\nu}_e$ 's discovered in the KamLAND experiment is due to  $\bar{\nu}_e \to \bar{\nu}_\mu$  and  $\bar{\nu}_e \to \bar{\nu}_\tau$  transitions. At  $\theta_{23} = \pi/4$  the probabilities of these transitions are equal.

At the Neutrino 2004 conference a new important result of the KamLAND collaboration was reported: the significant distortion of the spectrum of the reactor  $\bar{\nu}_e$  was observed [1].

As it is well known, in the KamLAND experiment the  $\bar{\nu}_e$ 's from many reactors in Japan and Korea are detected via the observation of  $e^+$  and n produced in the reaction

$$\bar{\nu}_e + p \to e^+ + n \tag{1}$$

In the paper [1] it is written "We present an improved measurement of the oscillations between first two neutrino families....."

In the framework of the standard three neutrino mixing

$$\nu_{lL} = \sum_{i=1}^{3} U_{li} \nu_{il} \tag{2}$$

 $(U^{\dagger}U = 1, \nu_i)$  is the field of neutrino with mass  $m_i$ ) we will consider here neutrino oscillations in the KamLAND experiment.

The transition probabilities of neutrino and antineutrinos can be presented in the form (see [2])

$$P(\nu_l \to \nu_{l'}) = |\delta_{ll'} + \sum_{i=2,3} U_{l'i} U_{li}^* \left( e^{-i\Delta m_{1i}^2 \frac{L}{2E}} - 1 \right)|^2$$
 (3)

and

$$P(\bar{\nu}_l \to \bar{\nu}_{l'}) = |\delta_{ll'} + \sum_{i=2,3} U_{l'i}^* U_{li} \left( e^{-i\Delta m_{1i}^2 \frac{L}{2E}} - 1 \right)|^2$$
 (4)

Here L is the source-detector distance, E is the neutrino energy and  $\Delta m_{1i}^2 = m_i^2 - m_1^2$ .

From the analysis of the existing neutrino oscillation data two important features of the neutrino mixing emerged:

1. 
$$\Delta m_{12}^2 \ll |\Delta m_{13}^2|$$

2. 
$$|U_{e3}|^2 = \sin^2 2\theta_{13} \ll 1$$

It follows from 1. and 2. that the dominant transition in the atmospheric range of L/E is  $\nu_{\mu} \to \nu_{\tau}$  ( $\bar{\nu}_{\mu} \to \bar{\nu}_{\tau}$ ). For the  $\nu_{\mu}$  ( $\bar{\nu}_{\mu}$ ) survival probability from (3) and (4) we find

$$P(\nu_{\mu} \to \nu_{\mu}) = P(\bar{\nu}_{\mu} \to \bar{\nu}_{\mu}) \simeq 1 - 2|U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2) \left(1 - \cos\Delta m_{13}^2 \frac{L}{2E}\right). (5)$$

In the approximation  $\sin^2 2\theta_{13} \to 0$  we have

$$U_{\mu 3} = \sin \theta_{23}, \quad U_{\tau 3} = \cos \theta_{23}.$$
 (6)

Thus, in the atmospheric range of L/E we obtain

$$P(\nu_{\mu} \to \nu_{\mu}) = P(\bar{\nu}_{\mu} \to \bar{\nu}_{\mu}) \simeq 1 - \frac{1}{2} \sin^2 2\theta_{23} (1 - \cos\Delta m_{13}^2 \frac{L}{2E}).$$
 (7)

From the analysis of the Super-Kamiokande atmospheric neutrino data the following best-fit values of the parameters were found [3]

$$\sin^2 2\theta_{23} = 1; \quad \Delta m_{13}^2 = 2 \cdot 10^{-3} \,\text{eV}^2$$
 (8)

For the probability of the transition  $\bar{\nu}_e \to \bar{\nu}_l$  in the KamLAND range of L/E from (4) (in the approximation  $\sin^2 2\theta_{13} \to 0$ ) we find

$$P(\bar{\nu}_e \to \bar{\nu}_l) = |\delta_{el} + U_{l2}^* U_{e2} \ (e^{-i\Delta m_{12}^2 \frac{L}{2E}} - 1)|^2$$
(9)

From this expression we obtain

$$P(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1 - 2|U_{e2}|^2 (1 - |U_{e2}|^2) (1 - \cos\Delta m_{12}^2 \frac{L}{2E}).$$
 (10)

Further from the unitarity of the mixing matrix we have

$$U_{e1} \simeq \cos \theta_{12}, \quad U_{e2} \simeq \sin \theta_{12}.$$
 (11)

Thus, in the KamLAND range of L/E the  $\bar{\nu}_e$  survival probability is given by the expression

$$P(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1 - \frac{1}{2} \sin^2 2\theta_{12} (1 - \cos\Delta m_{12}^2 \frac{L}{2E}).$$
 (12)

From the analysis of the latest KamLAND data and solar neutrino data in [1] it was found

$$\Delta m_{12}^2 = (8.2_{-0.5}^{+0.6}) \cdot 10^{-5} \,\text{eV}^2; \, \tan^2 2\theta_{12} = 0.40_{-0.07}^{+0.09}. \tag{13}$$

The unitarity of the neutrino mixing matrix implies

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - (P(\bar{\nu}_e \to \bar{\nu}_\mu) + P(\bar{\nu}_e \to \bar{\nu}_\tau)) \tag{14}$$

From (9) for the probabilities of the transitions  $\bar{\nu}_e \to \bar{\nu}_\mu$  and  $\bar{\nu}_e \to \bar{\nu}_\tau$  we find the following expressions

$$P(\bar{\nu}_e \to \bar{\nu}_\mu) \simeq 2|U_{e2}|^2|U_{\mu 2}|^2 (1 - \cos \Delta m_{12}^2 \frac{L}{2E}).$$
 (15)

and

$$P(\bar{\nu}_e \to \bar{\nu}_\tau) \simeq 2|U_{e2}|^2|U_{\tau 2}|^2 (1 - \cos \Delta m_{12}^2 \frac{L}{2E}).$$
 (16)

The elements  $U_{\mu 2}$  and  $U_{\tau 2}$  are determined by the angles  $\theta_{12}$  and  $\theta_{23}$ . In fact, from the unitarity of the mixing matrix we have

$$\sum_{i=1,2} |U_{\mu i}|^2 | = \cos^2 \theta_{23}; \quad \sum_{i=1,2} |U_{\tau i}|^2 | = \sin^2 \theta_{23}$$
 (17)

Taking into account that the raws of the mixing matrix must be orthogonal we easily find

$$U_{\mu 2} = \cos \theta_{23} \cos \theta_{12}; \quad U_{\tau 2} = -\sin \theta_{23} \cos \theta_{12}.$$
 (18)

Thus, we have

$$P(\bar{\nu}_e \to \bar{\nu}_\mu) = \cos^2 \theta_{23} \frac{1}{2} \sin^2 \theta_{12} \left( 1 - \cos \Delta m_{12}^2 \frac{L}{2E} \right). \tag{19}$$

and

$$P(\bar{\nu}_e \to \bar{\nu}_\tau) = \sin^2 \theta_{23} \frac{1}{2} \sin^2 \theta_{12} \left( 1 - \cos \Delta m_{12}^2 \frac{L}{2E} \right). \tag{20}$$

From (14), (19) and (20) we find the following relations between transition probabilities in the KamLAND range of L/E:

$$P(\bar{\nu}_e \to \bar{\nu}_\mu) = \cos^2 \theta_{23} \left( 1 - P(\bar{\nu}_e \to \bar{\nu}_e) \right) \tag{21}$$

and

$$P(\bar{\nu}_e \to \bar{\nu}_\tau) = \sin^2 \theta_{23} (1 - P(\bar{\nu}_e \to \bar{\nu}_e))$$
 (22)

From these relations it follows that

$$P(\bar{\nu}_e \to \bar{\nu}_\tau) = \tan^2 \theta_{23} P(\bar{\nu}_e \to \bar{\nu}_\mu). \tag{23}$$

Therefore the observation of neutrino oscillations in the atmospheric (K2K) experiments and the unitarity of the neutrino mixing matrix imply that the disappearance of reactor  $\nu_e$ , observed in the KamLAND experiment, is due to  $\bar{\nu}_e \to \bar{\nu}_\mu$  and  $\bar{\nu}_e \to \bar{\nu}_\tau$  transitions. For  $\theta_{23} = \pi/2$  (the SK best-fit value) the probabilities  $P(\bar{\nu}_e \to \bar{\nu}_\mu)$  and  $P(\bar{\nu}_e \to \bar{\nu}_\tau)$  are equal. <sup>1</sup>

Thus, in the leading approximation neutrino oscillations observed in the atmospheric (K2K) experiments are driven by  $\Delta m_{13}^2$  and are oscillations between second and third neutrino families. Neutrino oscillations observed in the KamLAND (solar) neutrino experiments are driven by  $\Delta m_{12}^2$ . Due to the unitarity of the mixing matrix all three neutrino families are involved in the oscillations.

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## References

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<sup>&</sup>lt;sup>1</sup>For arguments in favor of this equality see [4]

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